JOURNAL OF MATERIALS SCIENCE 12 (1977) 1325–1337

Tensile strength of hybrid composites

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This paper presents an approximate, statistical analysis for the tensile strength of unidirectional hybrid composite materials consisting of two-dimensional arrays of alternating low elongation and high elongation fibres in a common matrix. Expressions for ineffective length and fibre strain concentration factor in hybrid composites are developed. The analysis identifies a number of important material properties that affect the failure process in hybrids: statistical fibre tensile failure strain characteristics, and fibre extensional moduli and cross-sectional areas. The influence of these parameters on the failure process is examined and differences between failure mechanisms in hybrids and composites containing only one type of fibre are considered. The analysis predicts that, in general, the failure strain of a hybrid should be different from those of composites reinforced with either of the "parent" fibres alone. It is found that the theory can explain the "hybrid effect" that has been observed by several authors: hybrids made by combining high elongation and low elongation fibres, such as graphite and glass, often display tensile failure strains greater than those of composites made from the low elongation fibres alone. Predicted failure strains are compared with available experimental data. Suggestions for future work in the area are presented.

1. Introduction

There is increasing interest in hybrid composites – materials made by combining two or more different types of fibres in a common matrix – because these materials offer a range of properties that cannot be obtained with a single kind of reinforcement. At the same time, material costs can be substantially reduced by a careful selection of reinforcing fibres.

Fibres such as graphite and boron have low ultimate strains and make fairly brittle composites. It has been shown [1–5] that by combining these fibres with glass or "Kevlar" 49 aramid fibres, that have higher failure strains, it is possible to improve impact resistance. Zweben [6] has shown that a [0/90] hybrid laminate made from unidirectional graphite tape and "Kevlar" 49 fabric also had greater "fracture toughness" than an all-graphite panel. The advantages of hybrids in a variety of structural applications have been discussed [6–13], and mechanical properties presented [6, 14–16].

In order to confidently use a material, we need to understand how it behaves under load. Conversely, we are reluctant to use materials whose behaviour we do not understand. Phillips has summarized some of the major concerns surrounding the behaviour of hybrid composites [17]. The most puzzling seems to be in the area of tensile failure. Specifically, Phillips mentions that under tensile loading, the straight-line portion of some graphite–glass hybrids extends beyond the failure strain level of all-graphite composites. He credits the first observation of this phenomenon (the "hybrid effect") to Hayashi [18].

Bunsell and Harris [19] observed the "hybrid effect" in their tests on unidirectional laminates made from separate layers of high modulus graphite and glass fibres. The effect was attributed to residual compressive thermal strains in the graphite. They reported that the residual compressive strain in a single graphite/epoxy layer sandwiched between two layers of glass/epoxy was 0.00029, about 10% of the graphite/epoxy tensile failure strain of 0.0026. However, the mean strain at first break in hybrid specimens with layers bonded together was 0.0048, which is 0.0022 greater than

TABLE I Tensile failure strains of "Thornel" 300 "Kevlar" 49 and hybrid unidirectional composites

Reinforcement	Tensile failure strain		Ratio of mean strain to that of graphite composite
	Mean (%)	Coefficient of variation (%)	
"Thornel" 300 graphite	1.04	6.6	1.00
50 % "Thornel" 300/ 50% "Kevlar" 49	1.08	5.9	1.04
"Kevlar" 49	1.80	6.0	1.63

TABLE II Tensile failure strains of "Thornel" 300, "Kevlar" 49 and hybrid balanced fabric composites

Reinforcement	Tensile failure strain (%)	Ratio to Graphite strain
"Thornel" 300	0.71	1.00
50% "Thornel" 300/ 50% "Kevlar" 49	0.94	1.32
"Kevlar" 49	1.68	2.36

the graphite/epoxy failure strain. Only about 0.0003 of the increment is explained by residual thermal stresses. This leaves 0.0019 unaccounted for. That is, the hybrid tensile strain at first break is 80% greater than that of the high modulus graphite/epoxy alone, and only 10% of the increase can be explained by residual thermal stresses, according to the authors' data. Assuming these results to be valid, it appears we must look further for an explanation of the "hybrid effect".

We have observed this phenomenon in our own work with hybrids made by combining "Kevlar" 49 aramid and "Thornel" 300 graphite fibres in a Fiberite 934 epoxy matrix. Table I shows ultimate tensile strains for unidirectional hybrid specimens made from prepreg tapes which are made with alternating yarns of "Thornel" 300 graphites and "Kevlar" 49 aramid fibres. Since both yarns have very similar cross-sectional areas, the volumetric ratio of graphite-to-aramid is close to 50 : 50 [6]. We note that the hybrid failure strain is about 4% greater than that of the graphite control. We did not observe any departure from linearity before failure in these tests. That is, failure was catastrophic with no apparent pre-cracking in the graphite.

We also investigated the behaviour of composites reinforced with balanced hybrid fabrics [16]. By balanced, we mean that warp (longitudinal) and fill (transverse) constructions are similar. A composite made from balanced fabrics with all layers oriented in the same direction is analogous to a [0/90] laminate. Thy hybrid fabric was made by alternating yarns of "Kevlar" 49 and "Thornel" 300 in both warp and fill directions. As in the unidirectional composites, the ratio of graphite-to-"Kevlar" 49 was about 50 : 50. Table II shows tensile failure strains of the hybrid composites along with those of controls reinforced with graphite and aramid fabrics of identical construction. We note that the hybrid ultimate strain is substantially greater (32%) than that of the graphite control. As in the case of unidirectional composites, failure was catastrophic with no discernible pre-cracking of the graphite.

Our data indicate that the axial coefficient of thermal expansion of a typical unidirectional "Kevlar" 49/epoxy composite from 0 to 212° F is about -2×10^{-6} in. in.$^{-1}$ °F^{-1}. According to Hofer *et al.* [20], the corresponding value for a "Thornel" 300 composite is 0.01×10^{-6} in. in.$^{-1}$ °F^{-1} from 50 to 350° F. These values indicate that upon cooling, "Kevlar" 49 composites expand axially a small amount, and "Thornel" 300 composites undergo an almost negligible contraction. This means that the residual thermal strain introduced during specimen curing is positive (tensile) in the graphite. Therefore, it does not seem likely that the greater tensile failure strain can be accounted for by residual thermal strains. In fact, we would expect hybrids to have lower ultimate strains based on this effect alone. In any event, there would have to be a very large difference in coefficient of expansion between aramid and graphite fibres to explain, solely in terms of residual thermal strains, the 32% increase in failure strain observed in the case of hybrid fabric composites.

The hybrid effect is also evident in the data for "Kevlar" 49 combined with several graphite fibres presented in [15].

As we have seen, the "hybrid effect" has been observed independently by several investigators using different types of fibres and reinforcement configurations (collimated fibres and fabrics). Therefore, it is probably a real effect, and not a testing artifact. Furthermore, although data are not extensive, we tentatively conclude that residual thermal strains alone do not explain the effect. Therefore, if residual thermal strain is not the cause, the question as to what is, remains unsolved.

Perhaps the problem lies in the way we look at composite strength. We tend to think in terms of a unique, deterministic quantity, because of our ex-

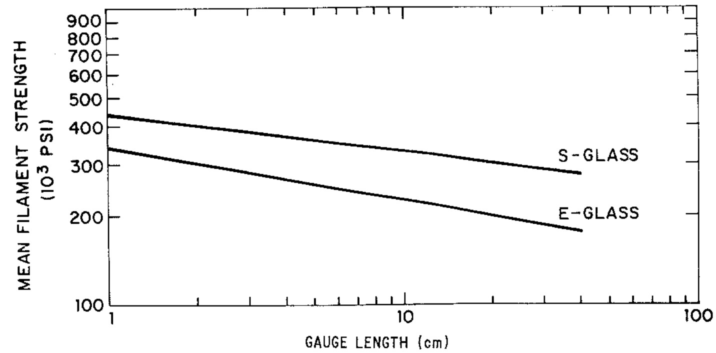

Figure 1 Variation with gauge length of the mean tensile strengths of E-glass and S-glass filaments (after Metcalfe and Schmitz [26]).

perience with metals. In fact, failure of composites under tensile load is a complex, statistical process involving fibre strength characteristics and matrix and interfacial properties [21–25]. If we look at tensile strength in this light, it is not surprising that hybrid composites have failure strains that differ from those of composites made from their "parent" fibres. Because hybrid composites are made from two or more different types of fibre which generally have very different mechanical properties, we can expect that failure processes in hybrids will differ from those of composites made with one type of fibre only. Therefore, it is reasonable to anticipate that their failure strains will differ.

2. Fibre strength properties

Before we begin the analysis, let us consider the strength characteristics of the major types of fibres used in composite materials. We emphasize that the strengths of these fibres are *statistical* in nature, and *not* deterministic. In general, fibre strength displays significant scatter at a fixed gauge length. Coefficients of variation (CV = standard deviation divided by mean) are typically 10 to 20%, and can be higher. In contrast, high-strength improved plow steel wire has a CV of 0.5 to 1%, or less. In addition to the scatter at fixed gauge lengths, we find that mean fibre strength depends on the gauge lengths at which they are tested. Mean strength decreases as gauge length increases. Therefore, it is incorrect to speak of "the" strength of a particular fibre. Fibres do not have a unique, deterministic strength. At best, we are only justified in referring to mean strength at a specified gauge length. These points about tensile strength also apply to fibre failure strain, which displays both scatter and gauge length dependence. Let us consider representative examples of the major renforcing fibres: E-glass, S-glass, boron, graphite, "Kevlar" 49 aramid.

Metcalfe and Schmitz [26] found that both E-glass and S-glass display significant scatter and length–strength dependence. Fig. 1 shows how the mean strengths of commercial E-glass and S-glass fibres vary with gauge length based on their data. Kies [27] also found a length–strength dependence in his tests on E-glass. He reported filament strength CVs of 24 and 28% at gauge lengths of 10 and 30 cm, respectively, indicating substantial scatter. The breaking strengths of one set of seventy-five 10 cm

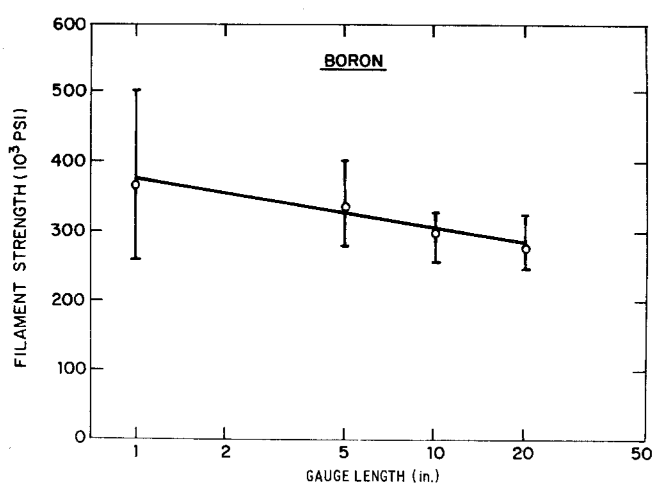

Figure 2 Variation with gauge length of the mean tensile strength of boron fibres (after Herring [28]).

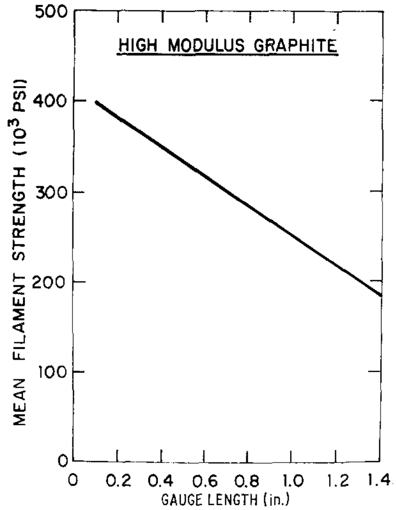

Figure 3 Variation with gauge length of the mean tensile strength of high modulus graphite fibres (after Diefendorf and Tokarsky [38]).

E-glass specimens ranged from a low of about 60 x 10^3 psi to a high of 300 x 10^3 psi.

If we assume that there is relatively little fibre-to-fibre variation in Young's modulus, which is probably valid for E-glass, and using a value of 10.5 x 10^6 psi for this quantity, we compute from the data in Fig. 1 that the tensile failure strain decreases from about 3.2% at a 1 cm gauge length to 2.2% at 10 cm. That is, glass fibres do not have a unique ultimate tensile strain. As in the case of tensile strength, it depends on gauge length.

Fig. 2 shows the scatter bands and mean strengths of boron filaments at four gauge lengths, as determined by Herring [28]. As in the case of E-glass and S-glass, there is no unique strength, or, by inference, ultimate strain.

Diefendorf and Tokarsky presented tensile strenghts data for high-modulus graphite fibres at various gauge lengths [38]. In Fig. 3 we have hand-fitted a straight line to their data. Again, we find a downward slope to the curve.

Fig. 4 shows how mean tensile strength and failure strain of "Kevlar" 49 filaments vary with gauge length. Within experimental error, the curves are parallel, and clearly show that both quantities depend on specimen length.

In the next section, we examine how the presence in a composite of two types of fibres with different moduli, cross-sectional areas, and failure strain characteristics affect the failure process in a hybrid composite.

3. Analysis

Our primary objective is to provide some insight into the mechanisms of failure in a material reinforced with two types of fibres that have different stiffness and strain characteristics. To do this, we construct idealized models that, hopefully, incorporate the major phenomena involved. The validity of the assumptions is open to challenge, but, in the long run, it is only by comparing the predicted results with experimental data that the usefulness of the analysis can be judged.

For simplicity we consider a hybrid composite of axial length L made from a single layer of fibres, as shown in Fig. 5. High modulus fibres with relatively low tensile strains are alternated with lower modulus fibres that have higher strains to break. The *low elongation* fibres are denoted by LE and subscript 1 and the *high elongation* fibres by HE and subscript 2. The respective moduli and cross-sectional areas of the fibres are denoted by E_1, A_1 and E_2, A_2. The total number of fibres in the composite is N, of which $N/2$ are LE, and $N/2$ are HE fibres.

In practice, hybrids are generally made by combining yarns of different types of fibres, rather than single filaments as we have assumed in our model. In order to handle this case, we assume that we can treat each impregnated yarn as if it were an individual fibre. The effective fibre area is equal to the total cross-sectional area of the fila-

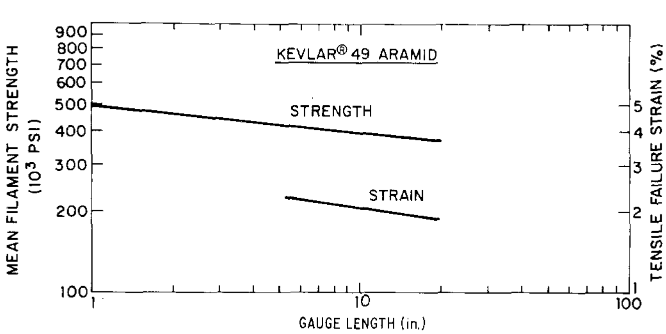

Figure 4 Variation with gauge length of the mean tensile strength and failure strain of "Kevlar" 49 aramid fibre.

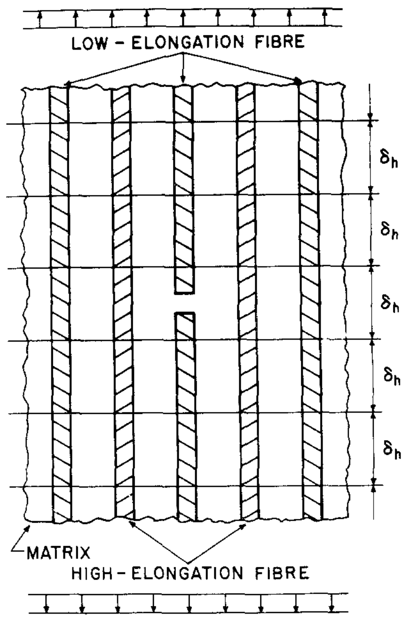

Figure 5 Model for tensile strength analysis of hybrid composites.

ments in the yarn. In this case, instead of using the statistical strength characteristics of the individual filaments, we use those of the impregnated strand. We argue that impregnated yarns are essentially small composites and fail by fibre break propagation, as described in [21, 22]. Fibre break propagation in the LE fibres will be arrested when the crack reaches the HE fibres. This is undoubtedly a major assumption. However, consideration of the failure processes in the individual yarns would increase the complexity of the analysis considerably, and as a first approximation, we believe the simplification is justified. In the remainder of the report we will use the term fibres, with the understanding that it represents both fibres and yarns.

Our analysis from this point on follows the procedures developed in [22]. We assume that the fibres support all of the applied load. As the composite is loaded, fibres break randomly throughout the material. Because the LE fibres have significantly lower ultimate strains, at a given load we expect to find many more breaks in these fibres than in the HE fibres. Because of this, we will focus our attention on what happens in the vicinity of random breaks in the LE fibres.

Since we are dealing with fibres of different moduli and cross-sectional areas, it is more convenient to consider composite strain as the independent variable, rather than stress which was used in [22].

When an LE fibre breaks at a strain level ϵ, the two HE fibres adjacent to it are subjected to a strain concentration of intensity $k_h\epsilon$, where k_h is the strain concentration factor for the hybrid material associated with a single broken LE fibre. The quantity k_h is analogous to the load concentration factor for a single broken fibre defined as k_1 in [31]. Since, in our simple model, axial fibre stress depends only on axial strain, k_h is also a fibre stress concentration factor for the hybrid. That is, the stress in the HE fibre next to a broken LE fibre increases from $E_2\epsilon$ to $k_hE_2\epsilon$ due to the break. Note that the strain concentration factor associated with the increase in strain in a LE fibre due to fracture of an HE fibre is generally different from k_h. This is discussed later.

It was shown [21] that the stress (and strain) perturbation arising from a fibre break is localized. The axial distance over which the stress is perturbed was defined in that work as the *ineffective length*, because the broken fibre is ineffective in carrying load over this portion of its length. In the present case, we denote the ineffective length associated with a broken LE fibre in the hybrid by δ_h.

We now consider the hybrid to be composed of a series of layers whose axial dimension is δ_h (Fig. 5). The number of layers is $M_h = L/\delta_n$. This is essentially the geometric model of Rosen [21] which, in turn, was an adaptation to composites of a general material strength model proposed by Gücer and Gurland [29].

The strain concentration in the HE fibres in the vicinity of broken LE fibres increases the probability that one or more of them will break in these regions. Therefore, every scattered break in an LE fibre can be thought of as a nucleus for fibre break propagation. As the strain in the composite is increased, the number of scattered LE fibre breaks increase, as does the strain level in the overstressed regions of HE fibres adjacent to LE fibre breaks. Eventually, the HE fibres will start to break at the points of strain concentration.

Following the approach used in [25], we hypothesize that the strain level at which the first overstressed fibre is expected to break is a lower bound on the ultimate strain of the hyrbid composite. That is, we postulate that failure results from the propagation of fibre breaks caused by localized strain concentrations. Therefore, the strain at which the first overstressed HE fibre is expected to break represents a lower bound on the strain associated with this mode of failure.

We now assume that the cumulative distribution functions for the failure strains of the LE and HE fibres of length l are given, respectively, by Weibull distributions of the form

$$F_1(\epsilon) = 1 - \exp(-pl\epsilon^q)$$
$$F_2(\epsilon) = 1 - \exp(-rl\epsilon^s) \quad (1)$$

where p, q and r, s are Weibull parameters of the respective distributions. We note that the Weibull distribution explicitly includes fibre length. This is one of its major features.

The derivation of the expression for ϵ_{2h} the composite strain at which the fracture of the first overstressed HE fibre in the hybrid is expected, is given in Appendix 1. We present the final result here.

$$\epsilon_{2h} = [NLpr\delta_h(k_h^s - 1)]^{-1/(q+s)} \quad (2)$$

For a composite reinforced with N fibres, all of which are of the same material, say the LE fibres, the equivalent expression is

$$\epsilon_2 = [2NLp^2\delta(k^q - 1)]^{-1/2q} \quad (3)$$

where k and δ are the strain concentration factor and ineffective length for the LE fibre composite, respectively. Note that k is identical to the load concentration factor k_1 used in [22]. The factor 2 appears in Equation 3 but not in Equation 2 because there are half as many LE fibres in the hybrid as there are in the all-LE fibre specimen.

We have obtained the above closed form solutions for ϵ_{2h} and ϵ_2 by expanding exponentials in Taylor series and keeping first terms only. These expressions replace the transcedental equation for the lower bound, Equation 4, in [22]*. By expanding that equation, we can obtain a closed form solution for the stress, σ_2, at which the first multiple break in a composite is expected. Assuming that the fibre tensile strength distribution is of the form $F(\sigma) = 1 - \exp(-\alpha l\sigma^\beta)$, we obtain the following closed form expression for the lower bound on the tensile strength of a composite in which fibres are in a square array

$$\sigma_2 = [4NL\delta\alpha^2(k_{1s}^\beta - 1)]^{-1/2\beta}$$
$$= \alpha_1[4NL\delta(k_{1s}^\beta - 1)]^{-1/2\beta} \quad (4)$$

where $\alpha_1 = \alpha^{-1/\beta}$, and k_{1s} is the fibre stress concentration factor associated with a single broken fibre in a square array.

For a two-dimensional (planar) array, as in the case of the material under study in this paper, the lower bound is given by

$$\sigma_2 = \alpha_1[2NL\delta(k_1^\beta - 1)]^{-1/2\beta}. \quad (5)$$

We are now in a position to write an expression for R_ϵ, the ratio of the lower bounds on failure strain of a hybrid to that of a composite of the same length that contains only LE fibres:

$$R_\epsilon = \epsilon_{2h}/\epsilon_2 = \frac{[prL\delta_h(k_h^s - 1)]^{-1/(q+s)}}{[2p^2L\delta(k^q - 1)]^{-1/2q}}. \quad (6)$$

We see that the ratio of failure strains depends on: LE and HE fibre tensile failure strain Weibull distribution parameters p, q and r, s; specimen length; ineffective lengths δ and δ_h; strain concentration factors k and k_h. Approximate expressions for ineffective lengths and strain concentration factors are derived by use of another approximate model based on the approach of [30]. Appendix 2 presents the details of the analysis: only the final expressions are given here.

$$\delta = 1.531\left(\frac{E_1A_1d}{gh}\right)^{1/2} \quad (7)$$

$$\delta_h = \frac{2}{\rho^{1/2}}\left(\frac{E_1A_1d}{Gh}\right)^{1/2}\frac{m_2^2 - m_1^2}{m_1(2-m_1^2) - m_2(2-m_2^2)} \quad (8)$$

$$k = 1.293 \quad (9)$$

$$k_h = 1 + \frac{m_2 - m_1}{m_1(2-m_1^2) - m_2(2-m_2^2)} \quad (10)$$

where E_1A_1 is the extensional stiffness of LE fibres, E_2A_2 the extensional stiffness of HE fibres, G the matrix shear modulus, h the matrix thickness, d the fibre spacing (Fig. 6), $\rho = E_1A_1/(E_2A_2)$ = ratio of fibre extensional stiffnesses, and

$$m_{1,2} = \left(\frac{\rho + 1 \pm (\rho^2 + 1)^{1/2}}{\rho}\right)^{1/2}.$$

We note that δ and k can be derived from the general expression for δ_h and k_h, respectively, by setting $\rho = 1$. The value of k given by the approximate model, 1.293, is quite close to that obtained with a different model by Hedgepeth, 1.333 [31].

We observe that the expressions for δ_h and k_h can be greater or less than the corresponding values for a LE fibre composite, depending on

* Equation 4 in [22] should read: $MNF(\sigma)p_{1/2} = 1$.

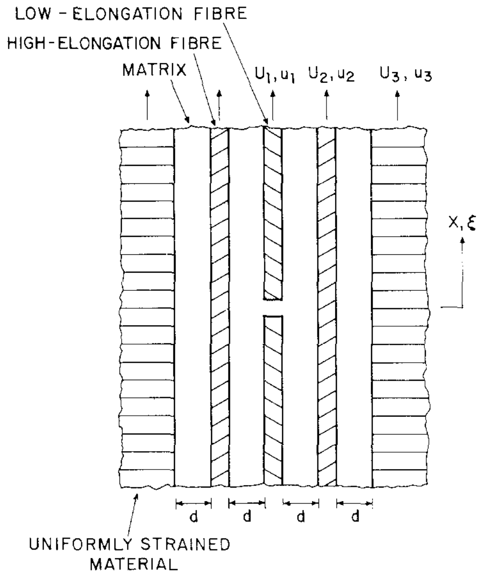

Figure 6 Model for determination of hybrid ineffective length, $\delta_{\rm h}$, and strain concentration factor, $k_{\rm h}$.

whether the ratio of fibre extensional stiffness, ρ, is greater or less than one. Since ρ is independent of fibre strength properties, we see that, at least in principle, it is possible to construct hybrids with lower bounds on failure strain that are lower or higher than that of an LE fibre composite.

When the HE and LE fibres display a similar amount of scatter, the expression for the ratio of lower bound strains, Equation 6 can be greatly simplified. Let us assume that both fibres have the same tensile failure strain coefficient of variation, $\bar{c}$, which is defined as standard deivation divided by mean. (We note that coefficient of variation is independent of gauge length for the Weibull distribution). It can be shown that for this case, $q = s$ and Equation 6 reduces to

$$R_\epsilon = \left(\frac{r}{p}\right)^{-(1/2q)} \left[\frac{\delta_{\rm h}(k_{\rm h}^q - 1)}{2\delta(k^q - 1)}\right]^{-1/2q}. \quad (11)$$

We now observe that $\bar{\epsilon}_1(l)$ and $\bar{\epsilon}_2(l)$, the mean strains of LE and HE fibres tested at a gauge length l, given by

$$\bar{\epsilon}_1(l) = (pl)^{-1/q}\Gamma\left(1 + \frac{1}{q}\right) \quad (12)$$

$$\bar{\epsilon}_2(l) = (rl)^{-1/q}\Gamma\left(1 + \frac{1}{s},\right)$$

where Γ is the gamma function.

Since $q = s$, Equation 11 can be rewritten as

$$R_\epsilon = \left[\frac{\bar{\epsilon}_2}{\bar{\epsilon}_1}\right]^{1/2} \left[\frac{\delta_{\rm h}(k_{\rm h}^q - 1)}{2\delta(k^q - 1)}\right]^{-1/2q}, \quad (13)$$

i.e. for the special case in which LE and HE fibres have the same coefficient of variation, the ratio of hybrid lower bound strains to that of LE composites is directly proportional to the square root of the ratio of fibre strains at an arbitrary gauge length, l, multiplied by a function of ineffective lengths, stress concentration factors and the Weibull parameter q, which is inversely related to fibre coefficient of variation.

When the fibre coefficient of variation is small, say about 5%, or less (which corresponds to $\beta > 25$), and $k_{\rm h} \geqslant k$, we can introduce another simplification that makes it easier to assess the importance of the several parameters. For this case, $k^q - 1 \cong k^q$ and $k_n^q - 1 \cong k^q$, and Equation 13 reduces to

$$R_\epsilon = 2^{1/2q}\left(\frac{\bar{\epsilon}_2}{\bar{\epsilon}_1}\right)^{1/2}\left(\frac{k}{k_{\rm h}}\right)^{1/2}\left(\frac{\delta}{\delta_{\rm h}}\right)^{1/2q}. \quad (14)$$

Equation 14 shows that the ratio of lower bound strains depends strongly on the mean fibre tensile failure strains and strain concentration factors, while the influence of ineffective length is relatively small, since q is quite large. The limitations on Equation 14 should be kept in mind. We have assumed that LE and HE fibres have the same failure strain coefficient of variation, and this is less than about 5%.

In the next section, we compute ratio of lower bounds on failure strains and compare these with experimental data.

4. Comparison with experimental data

We consider two cases: "Kevlar 49/"Thornel" 300 [6] and a high modulus graphite/E-glass system [19].

As discussed in Section 1, the "kevlar" 49/ "Thornel" 300 hybrids were made by alternating yarns of the two materials and not individual filaments. Therefore, we assume that we can treat the individual impregnated yarns as if they were fibres. Unfortunately, we do not have strength and failure strain data for impregnated yarns of the two materials, but we do have data for unidirectional composite tensile coupons. We assume that these values are close enough to those of impregnated yarns to use for the present study. This is equivalent to assuming that the effect of specimen volume is small. Table I presents tensile coupon data. We note that the coefficients of variation for

"Kevlar" 49 and "Thornel" 300 composites are similar and relatively small, so that we are justified in using the simplified expression for R_ϵ presented in Equation 14. Using Equation 10 we find that $k_h = 1.462$ which is only about 13% greater than the corresponding value for an LE composite of 1.293. The ratio of hybrid ineffective length-to-LE ineffective length is found to be 1.573 : 1.531 = 1.03 by use of Equations 9 and 10. We assume that both the fibres have a fibre strain coefficient of variation of about 6% which approximately corresponds to a value of 20 for the Weibull parameter q. Substituting these values in Equation 14, we find that

$$R_\epsilon = (1.017)(1.279)(0.940)(0.999) = 1.22,$$

where the four members in parentheses correspond to the four terms on the right-hand side of the original equation. By examining the contributions of these four quantities, we gain some insight into the effects of the major parameters we have included in our analysis. The first term arises because there are twice as many LE fibres in an all-LE fibre composite as there are in the hybrid. Since the LE fibres represent sources of defects that can cause fibre break propagation, the analysis predicts that strength decreases with increasing number of fibres. This is essentially a volume effect. The second term, which is the dominant one, reflects the predicted increase in strain related to the ratio of mean fibre strains. Note that it is a square root dependence. The higher fibre strain concentration factor for hybrids causes the third term to be less than unity. Although the ineffective length of the hybrid is greater than that of the all-LE fibre composite, its effect on strength is extremely small, since the ratio of ineffective lengths appears to the $1/2q$ power.

The experimental ratios of hybrid-to-graphite strains were found to be 1.04 for unidirectional composites and 1.31 for balanced fabrics. The predicted value falls just about in the middle of these two values.

Bunsell and Harris tested composites consisting of one or two layers of high modulus graphite/epoxy sandwiched between two layers of E-glass/epoxy. Failure strains for the parent materials were found to be 0.0026 and 0.0175, respectively. The respective graphite and glass composite moduli were 142 and 41 GN m^{-2}. No data were given on coefficients of variation. We assume they were the same as we found for "Kevlar" 49 and "Thornel" 300, about 6%.

Although the specimen geometry of these layered hybrids is different from the array of single fibres we analysed, it is instructive to see what type of behaviour the model predicts for these materials. Using the equations as discussed above, we find that $k_h = 1.753$, and the ratio of ineffective lengths $\delta_h/\delta = 1.627/1.531 = 1.06$. Substituting in Equation 14 with $q = 20$, we find that

$$R_\epsilon = (1.017)(2.59)(0.859)(0.998) = 2.26.$$

Experimental strain ratios were 1.31 for the case of two layers of graphite in the middle and 1.83 for the case of one layer in the middle. Although the predicted ratio is greater than either value, it is perhaps significant that both experimental ratios are rather large. We observe further that the experimental value for the three-layer material, which more closely fits the assumed model, in which LE and HE fibres alternate, is the greater one.

5. Conclusion

In this paper we have attempted to gain some insight into the behaviour under tensile loading of hybrid composites reinforced with alternating low-elongation and high-elongation fibres which have different tensile failure strain characteristics. To do this we have constructed two simple, highly idealized models: one to study statistical failure mechanisms, and the other to obtain hybrid strain concentration factors and ineffective lengths. The analysis predicts than for some systems, the lower bound strain on fibre break propagation is higher for hybrids that it is for LE fibre composites. Considering the many simplifying assumptions made in developing the analysis, agreement with experimental data is not unreasonable.

In essence, the analysis predicts that the introduction of HE fibres in an LE fibre composite raises the strain level required to propagate fibre breaks because the HE fibres behave like crack arrestors on a micromechanical level. The use of high-elongation materials to arrest propagating cracks in a composite on a macroscopic level has been studied by Eisenman and Kaminski [32], among others, and has been shown to be effective for some configurations.

This analysis presented here is a very simplified approach to a complex problem. Undoubtedly, more sophisiticated models could contribute more to our understanding of the processes involved.

For example, the approach developed by Phoenix [33] provides a strength distribution function for composites associated with an assumed failure mechanism, rather than the simple lower bound expression developed here. In addition, the many simplifying assumptions made in our development need to be considered in greater detail.

There may well be other approaches that can explain the hybrid effect. For example, Kelly has suggested that it might be worthwhile to consider the problem on an energy basis [34]. This approach was found [35, 36] to provide an explanation for the higher matrix fracture strains of brittle matrix composites.

We believe that hybrid composites will assume increasing importance in the coming years. It is important that we understand how they behave to provide for their intelligent and reliable application.

Appendix 1. Lower bound on composite failure strain

In this section, we develop expressions for the lower bounds on composite strain associated with the fibre break propagation mode of failure for hybrid and low elongation fibre composites. We use the approach previously developed [22, 25].

Consider a composite consisting of a planar (two-dimensional) array of alternating HE and LE fibres, as shown in Fig. 5. We assume that fibres carry only axial load, and the strain in a given fibre is constant across the thickness of that fibre at any cross-section. Because we are primarily concerned with resin-matrix composites, the axial load supported by the matrix is small compared to that of the fibres, and is neglected.

We assume that the tensile failure strain distributions of the LE and HE fibres are given, respectively, by Weibull distributions of the form

$$\begin{aligned} F_1(\epsilon) &= 1-\exp(-pl\epsilon^q) \\ F_2(\epsilon) &= 1-\exp(-rl\epsilon^s) \end{aligned} \qquad \text{(A1)}$$

where l is fibre length and p and q, and r and s are Weibull parameters of the respective distributions. Where yarns are used, we treat them as if they were individual fibres, and use the impregnated yarn failure strain properties to define $F_1(\epsilon)$ and $F_2(\epsilon)$.

We assume that the properties of the failure strain distributions are such that under the range of applied strains of interest, the number of scattered HE fibre breaks are small compared to the number of LE fibre breaks, and we ignore the former. Let the total number of fibres in the composite be N, equally divided between LE and HE fibres. We assume that the composite, whose length is L, is loaded in axial tension. Away from the boundary region near the loaded ends of the composite, the axial strain in the HE and LE fibres is equal and is the uniform composite strain, ϵ, except in the vicinity of fibre breaks. Near a broken LE fibre, the stress in both the broken and surrounding fibres is perturbed over an axial distance, δ_h, the ineffective length in the hybrid associated with the fracture of an LE fibre. An expression for δ_h is derived in Appendix 2. We note that the ineffective length arising from an HE fibre break in the hybrid is generally different from δ_h. Following the approach of Rosen [21], we assume that the composite is composed of M_h layers whose axial length is δ_h, where $M_h = L/\delta_h$. The stress concentration arising from a broken fibre is assumed to affect only the two adjacent fibres in the same layer.

When the composite is subjected to a strain ϵ, the expected number, X_{1h}, of scattered fibre breaks in the N/2 LE fibres is

$$X_{1h} = \tfrac{1}{2}M_hNF_1(\epsilon), \qquad \text{(A2)}$$

where δ_h is the appropriate fibre length to use in $F_1(\epsilon)$.

Next to each LE fibre, there are two HE fibres subjected to a strain concentration $k_h\epsilon$, where k_h is the strain concentration factor associated with an LE fibre break in the hybrid. An expression for k_h is developed in Appendix 2. As in the case of ineffective length, the strain concentration factor associated with an LE fibre break in the hybrid is different from that of an HE fibre break. We assume that the strain concentration is constant in the two adjacent fibres throughout the entire length of the layer in which the break occurs.

We note that in our model, k_h is also a stress concentration factor. It describes the increase in stress in an overstressed HE fibre relative to the stress in undisturbed HE fibres, which is $E_2\epsilon$, where E_2 is the Youngs, modulus of the HE fibres. That is, the stress increases from $E_2\epsilon$ to $k_hE_2\epsilon$.

Now consider what happens to an HE fibre that is initially at a strain level ϵ, and is suddenly subjected to a strain $k_h\epsilon$. The probability that it will fail due to the strain increment $(k_h-1)\epsilon$, assuming it has not failed at the strain level, ϵ, is

$$P_{\mathrm{h}} = \frac{F_2(k_{\mathrm{h}}\epsilon) - F_2(\epsilon)}{1 - F_2(\epsilon)}. \quad \text{(A3)}$$

The appropriate fibre length for use with Equation A3 is δ_{h}, the hybrid ineffective length

The probability that at least one of the two overstressed fibres will break is

$$P_{2\mathrm{h}} = 1 - (1 - P_{\mathrm{h}})^2. \quad \text{(A4)}$$

The expected number of sites where a scattered LE fibre break is followed by the fracture of at least one of the adjacent overstressed fibres is

$$X_{2\mathrm{h}} = X_{1\mathrm{h}} P_{2\mathrm{h}}. \quad \text{(A5)}$$

As in [25], we propose that the fracture of one of the overstressed fibres is a lower bound on the stress at which fibre breaks will propagate, causing composite failure. The mathematical expression for this event it

$$X_{2\mathrm{h}}(\epsilon_{2\mathrm{h}}) = 1, \quad \text{(A6)}$$

where $\epsilon_{2\mathrm{h}}$ is the composite strain, associated with the fracture of the first overstressed HE fibre. Again, this is the proposed lower bound on hybrid failure associated with the fibre break propagation failure mode.

By making some simplifying assumptions, we can obtain a closed form solution for $\epsilon_{2\mathrm{h}}$. Composite failure occurs when the magnitude of the cumulative distribution functions is very small compared to unity. Therefore, we neglect $F_2(\epsilon)$ with respect to unity in the denominator of Equation A3. We also neglect products and squares of distribution functions in Equation A4. Based on these assumptions, Equation A6 reduces to

$$X_{2\mathrm{h}} = M_{\mathrm{h}} N F_1(\epsilon_{2\mathrm{h}})[F_2(k_{\mathrm{h}}\epsilon_{2\mathrm{h}}) - F_2(\epsilon_{2\mathrm{h}})] = 1. \quad \text{(A7)}$$

We now assume that the exponentials in Equation A1 can be adequately represented by the first term of their Taylor series expansions. That is

$$\begin{aligned} \exp(-pl\epsilon^q) &\cong 1 - pl\epsilon^q \\ \exp(-rl\epsilon^s) &\cong 1 - rl\epsilon^s. \end{aligned} \quad \text{(A8)}$$

Substituting these values in Equation A1 we obtain

$$\begin{aligned} F_1(\epsilon) &\cong pl\epsilon^q \\ F_2(\epsilon) &\cong rl\epsilon^s. \end{aligned} \quad \text{(A9)}$$

Making use of Equation A9 in conjunction with Equation A7 we arrive at the simplified, closed-form expression for the lower bound strain,

$$\epsilon_{2\mathrm{h}} = [NL\delta_{\mathrm{h}} pr(k_{\mathrm{h}}^s - 1)]^{-1/(q+s)}. \quad \text{(A10)}$$

Here, we have also made use of the relation $M_{\mathrm{h}} = L/\delta_{\mathrm{h}}$.

The equivalent expression for a composite of length L containing N low-elongation fibres is

$$\epsilon_2 = [2NL\delta p^2(k^q - 1)]^{-1/2q}. \quad \text{(A11)}$$

This expression is derived in the same way as Equation A10. The factor of 2 in the brackets arises in Equation A11 because there are twice as many LE fibres in this composite as there are in the hybrid. Here, δ and k are, respectively, the ineffective length and strain concentration factor for composites having only LE fibres. Expressions for these quantities are derived in Appendix 2.

Appendix 2 Analysis of ineffective length and strain concentration factor

In this section we derive expressions for ineffective length, δ_{h}, and strain concentration factor, k_{h}, in a hybrid composite associated with the fracture of an LE fibre. We use a simple model based on the one developed in [30].

The model used to develop expressions for k_{h}, and δ_{h} is shown in Fig. 6. We consider a broken LE fibre separated from two adjacent HE fibres by a matrix region of width d and thickness h with a shear modulus G. The HE fibres are flanked by a region of material in which, it is assumed, the strain is uniform and equal to the composite strain ϵ. Between the HE fibres and this uniformly strained material is a matrix region of width d and thickness h. The Young's moduli of the LE and HE fibres are E_1 and E_2, respectively, and their respective cross-sectional areas are A_1 and A_2. When the hybrid is made with alternating yarns, rather than single filaments, we use the total cross-setional areas of the filaments in the yarn to define A_1 and A_2. We assume that the fibres carry only axial stress, which is uniform across their cross-section. We neglect axial stress in the matrix and consider it to be a medium for transmission of shear stress alone. This is essentially a simplified shear lag model based on concepts proposed in [31]. For simplicity, we shall refer to the LE and HE fibres as fibres 1 and 2, respectively.

Based on these assumptions, we can now develop the equations of equilibrium for fibres 1 and 2. Since we have a mixed boundary value problem we work with fibre axial displacements which are denoted by U_1 and U_2, respectively. Our origin is in the cross-section of the fibre break. The model is symmetric about this plane. The axial displacement of the material in the region of uniform strain is $U_3 = \epsilon x$, where x is the axial distance from the plane of the origin.

Based on the above assumptions, the equations of equilibrium for fibres 1 and 2 are, respectively,

$$E_1 A_1 \frac{d^2 U_1}{dx^2} + \frac{2Gh}{d}(U_2 - U_1) = 0$$

$$E_2 A_2 \frac{d^2 U_2}{dx^2} + \frac{Gh}{d}(U_1 - 2U_2) = -\frac{Gh}{d}\epsilon x. \qquad \text{(A12)}$$

We require that as $x \to \infty$, the strains in the HE and LE fibre approach the composite strain ϵ. That is,

$$\lim_{x\to\infty} \frac{dU_1}{dx} = \lim_{x\to\infty} \frac{dU_2}{dx} = \epsilon. \qquad \text{(A13)}$$

At the origin, the stress in fibre 1 is zero, and by symmetry, the displacement in fibre 2 must be zero. Therefore,

$$\frac{dU_1(0)}{dx} = U_2(0) = 0. \qquad \text{(A14)}$$

It is convenient to express these equations in a non-dimensional form, as in [31]. To do this, we introduce dimensionless displacements u_1, u_2, and u_3 and a dimensionless axial distance ξ which are related to their corresponding dimensioned variables by

$$U_{1,2,3} = \left[\frac{E_2 A_2 d}{Gh}\right]^{1/2} \epsilon u_{1,2,3}$$

$$x = \left[\frac{E_2 A_2 d}{Gh}\right]^{1/2} \xi. \qquad \text{(A15)}$$

Using Equation A15, we find that $u_3 = \xi$.

We now introduce another dimensionless parameter, ρ, the ratio of fibre extensional stiffnesses, defined as

$$\rho = \frac{E_1 A_1}{E_2 A_2}. \qquad \text{(A16)}$$

We note that ρ can be greater, or less than unity. In general, we would expect it to be greater.

Using these dimensionless quantities, the equations of equilibrium, Equation A12, become

$$\rho \frac{d^2 u_1}{d\xi^2} + 2(u_2 - u_1) = 0$$

$$\frac{d^2 u_2}{d\xi^2} - 2u_2 + u_1 = -\xi \qquad \text{(A17)}$$

and the boundary conditions reduce to

$$\lim_{\xi\to\infty} \frac{du_1}{d\xi} = \lim_{\xi\to\infty} \frac{du_2}{d\xi} = 1$$

$$\frac{du_1(0)}{d\xi} = u_2(0) = 0. \qquad \text{(A18)}$$

The solution to this set of equations is

$$u_1 = \xi + (2 - m_1^2)Ce^{-m_1\xi} - (2 - m_2^2)Ce^{-m_2\xi}$$

$$u_2 = \xi + Ce^{-m_1\xi} - Ce^{-m_2\xi} \qquad \text{(A19)}$$

where

$$m_1 = \left(\frac{\rho + 1 + (\rho^2 + 1)^{1/2}}{\rho}\right)^{1/2}$$

$$m_2 = \left(\frac{\rho + 1 - (\rho^2 + 1)^{1/2}}{\rho}\right)^{1/2}$$

$$C = [m_1(2 - m_1^2) - m_2(2 - m_2^2)]^{-1}.$$

We observe that the equations of equilibrium and the boundary conditions for the problem considered here, that of a single broken LE fibre, are identical to those derived in [30] for the case of a notch in a unidirectional composite that cuts n fibres, except that here n is replaced by ρ.

The strain concentration factor associated with a broken LE fibre is defined as the ratio of strain in the adjacent HE fibres, fibres 2, at the origin to the applied composite strain ϵ. That is,

$$k_h = \frac{1}{\epsilon}\frac{dU_2(0)}{dx}. \qquad \text{(A20)}$$

In dimensionless form, this becomes

$$k_h = \frac{du_2(0)}{d\xi}. \qquad \text{(A21)}$$

The resulting expression for hybrid strain concentration factor is

$$k_h = 1 - (m_1 - m_2)C. \qquad \text{(A22)}$$

Since the governing equations are identical, this expression also given stress concentrations associated with a notch cutting n fibres in a composite that contains only one kind of fibre, which was derived in [30]. If we let $\rho = 1$, the problem reduces to the case of a single broken fibre in a composite containing only one kind of fibre. We denote this quantity by k, and the resulting value for the strain concentration factor obatined with Equation A22 is $k = 1.293$. As we discussed in the body of the report, k_h and k are also stress concentration factors, although in the case of a hybrid composite we must be careful how we interpret "stress concentration". In the case of a composite containing only one type of fibre, there is no confusion, and, in our simple model, strain concentration factor and stress concentration factor are identical. Therefore, we can compare the strain concentration factor obtained here for $\rho = 1$, which we found to be 1.293, with Hedgepeth's stress concentration factor for a single broken fibre which is 1.333 [31]. The difference is small, which lends support for the validity of this simple model.

We now turn our attention to ineffective length, δ_h. This quantity is a measure of the axial dimension over which the stress is perturbed in the vicinity of a broken LE fibre in the hybrid composite. We adopt the definition proposed by Friedman in [37]. Fig. 7 shows his concept schematically. We replace the stress distribution in the broken fibre by a step function that has the same average fibre stress. In our analysis, we can equivalently work with fibre strain. The requirement that average strains be equal implies that the areas between the strain distribution curves and the constant strain ϵ be equal. That is,

$$\frac{\delta_h}{2}\epsilon = \int_0^\infty \left(\epsilon - \frac{dU_1}{dx}\right) dx. \qquad (A23)$$

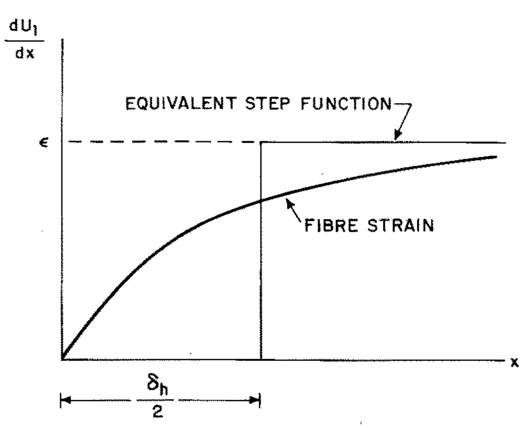

Figure 7 Definition of ineffective length (after Friedman [37]).

Using Equations A15 and A16, we can write this as

$$\delta_h = \frac{2}{\rho^{1/2}}\left[\frac{E_1 A_1 d}{Gh}\right]^{1/2} \int_0^\infty \left[1 - \frac{du_1}{d\xi}\right] d\xi.$$

The dimensionless displacement u is given in Equation A19. Performing the indicated integration, we find

$$\delta_h = \left[\frac{E_1 A_1 d}{Gh}\right]^{1/2} \bar{\delta}_h, \qquad (A24)$$

where

$$\bar{\delta}_h = \frac{2(m_2^2 - m_1^2)}{\rho^{1/2}[m_1(2-m_1^2) - m_2(2-m_2^2)]}. \qquad (A25)$$

The ineffective length for a composite reinforced with LE fibres only, δ, can be obtained from Equation A24 by setting ρ equal to unity. Performing this operation, we find

$$\delta = 1.531\left[\frac{E_1 A_1 d}{Gh}\right]^{1/2}. \qquad (A26)$$

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Received 29 October and accepted 2 December 1976.